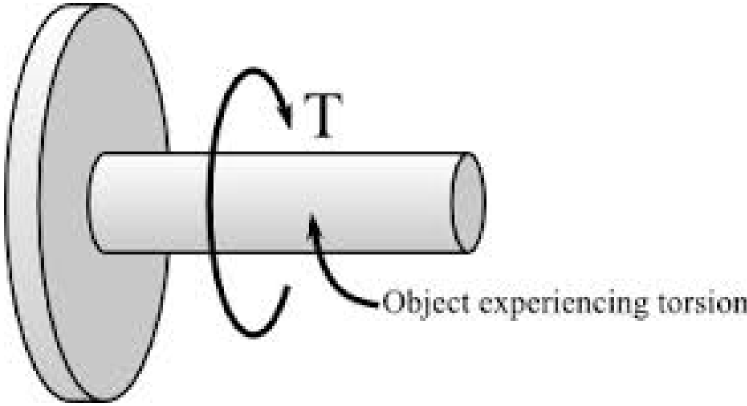
TORSION

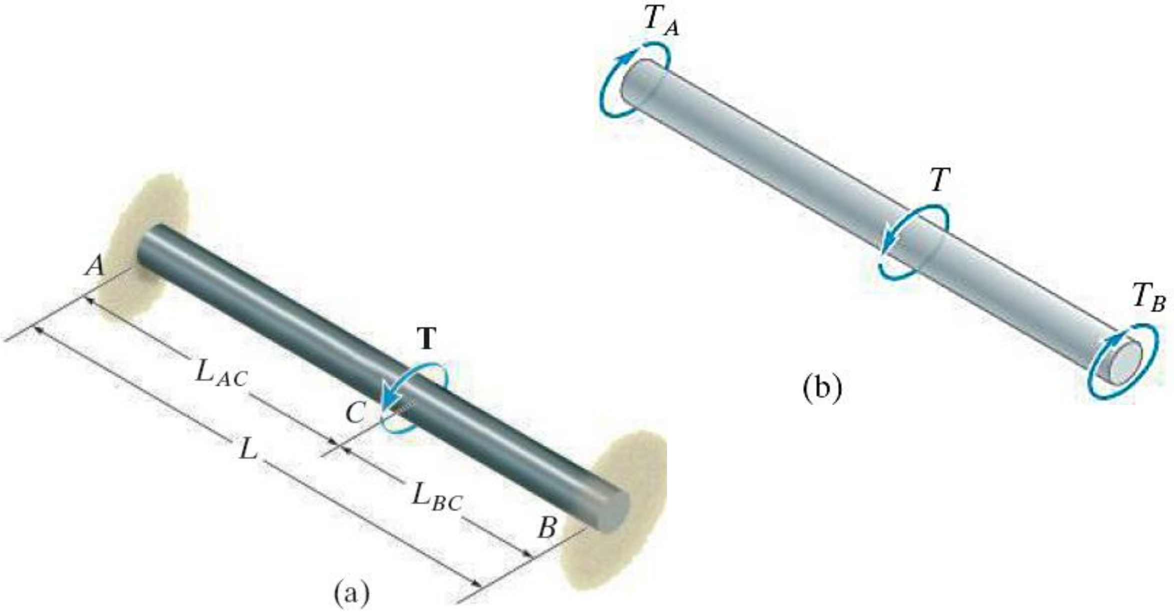
5
Torsion 181



5.5 Statically Indeterminate Torque-loaded Members

A torsion-ally loaded shaft may be classified as statically indet-erminate if the moment equation of equilibrium, applied about the axis of the shaft, is not adequate to determine the unknown torques acting on the shaft.

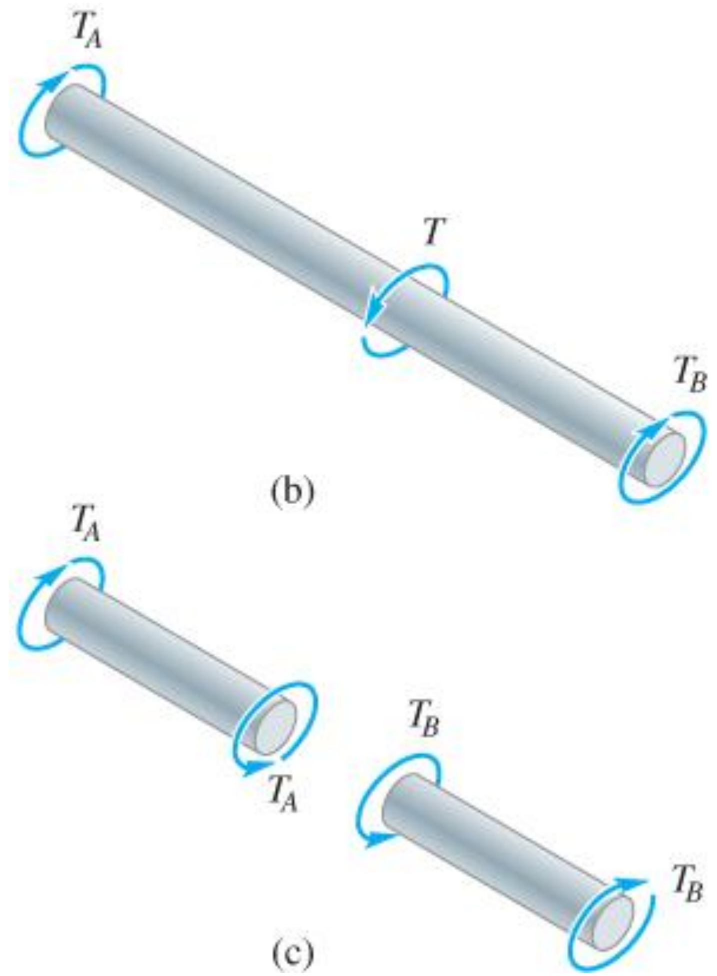
An example of this situation is shown in Fig. 5–22 *a* . As shown on the free-body diagram, Fig. 5–22 *b* , the reactive torques at the supports *A* and *B* are unknown. We require that



From free-body diagram, reactive torques at supports A and B are unknown, Thus,

$$\Sigma M_x = 0;$$

$$T - T_A - T_B = 0$$



Since problem is statically indeterminate, formulate the condition of compatibility; end supports are fixed, thus angle of twist of both ends should sum to zero

$$\phi_{A/B} = 0$$

Assume linear-elastic behavior, and using load-displacement relationship, $\phi = TL/JG$, thus compatibility equation can be written as

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0$$

Solving the equations simultaneously, and realizing that

$L = L_{AC} + L_{BC}$, **we get**

$$T_A = T\left(\frac{L_{BC}}{L}\right) \quad \text{and} \quad T_B = T\left(\frac{L_{AC}}{L}\right)$$

Procedure For Analysis

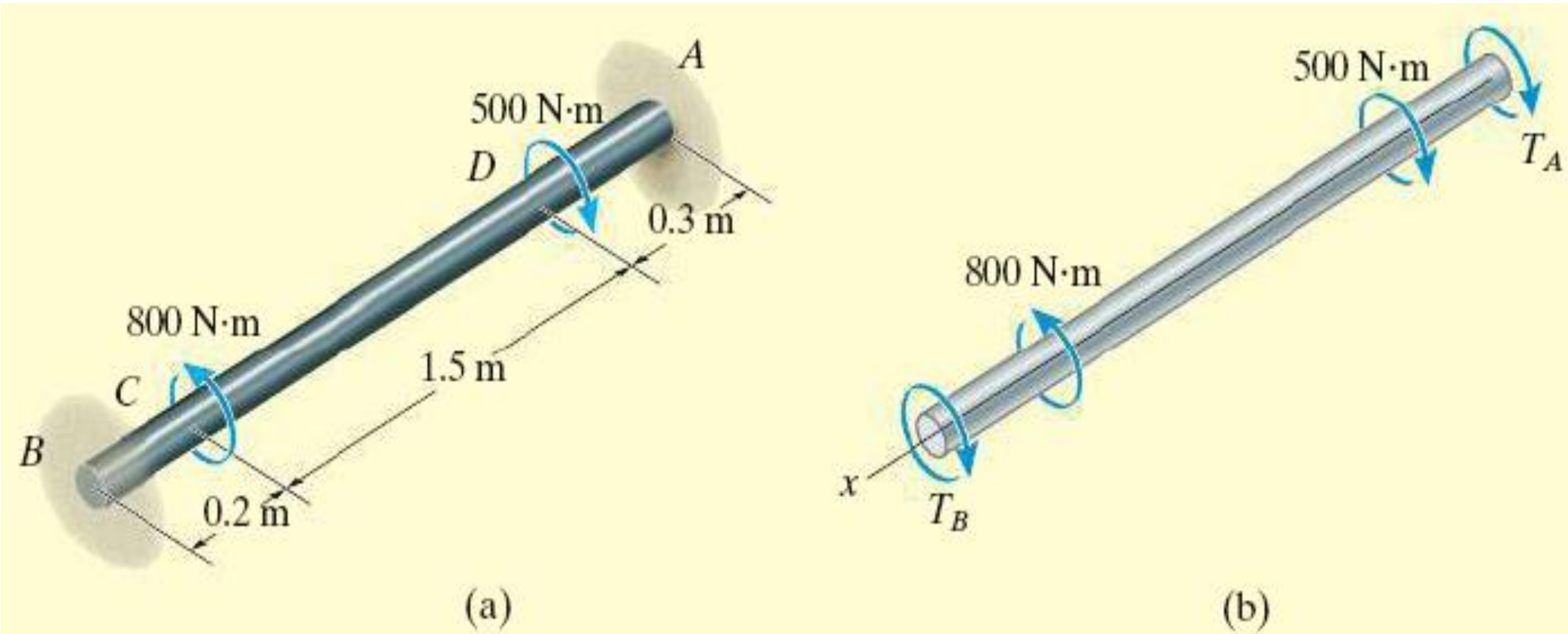
- **Equilibrium**
  - Draw a free-body diagram
  - Write equations of equilibrium about axis of shaft.

- **Compatibility**
  - Express compatibility conditions in terms of rotational displacement caused by reactive torques
  - Use torque-displacement relationship, such as $\phi = TL/JG$
  - Solve equilibrium and compatibility equations for unknown torques

TORSION

The shaft of this cutting machine is fixed at its ends and subjected to a torque at its center, allowing it to act as a torsional spring.

Ex:-The solid steel shaft shown in Fig. 5–23 *a* has a diameter of **20 mm**. If it is subjected to the two torques, determine the reactions at the fixed supports ***A*** **and** ***B*****.**



Strength of Materials: Second Class

SOLUTION

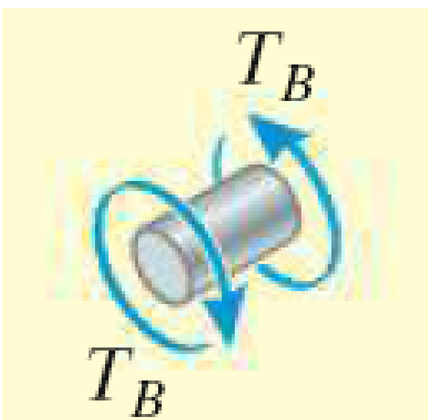
Equilibrium. By inspection of the free-body diagram, Fig. 5–23*b*, it is seen that the problem is statically indeterminate since there is only *one* available equation of equilibrium and there are two unknowns. We require

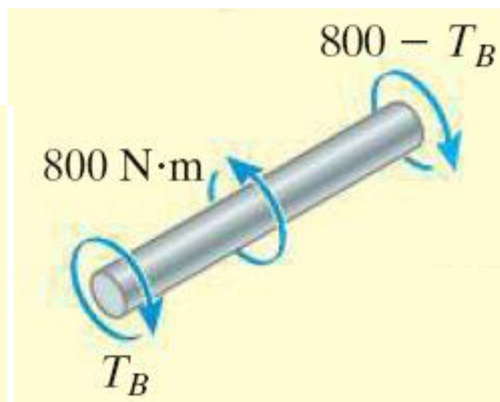
$$\Sigma M_x = 0; \qquad -T_B + 800\,\text{N}\cdot\text{m} - 500\,\text{N}\cdot\text{m} - T_A = 0 \qquad (1)$$

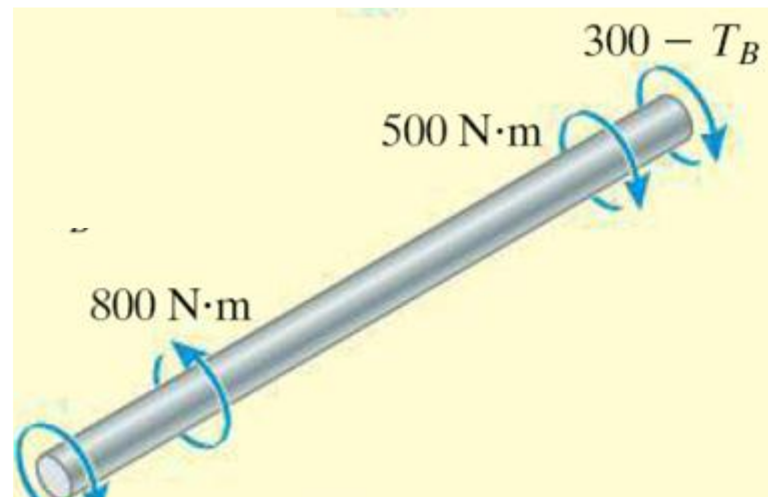
Compatibility. Since the ends of the shaft are fixed, the angle of twist of one end of the shaft with respect to the other must be zero. Hence, the compatibility equation becomes

$$\phi_{A/B} = 0$$

Load-Displacement. This condition can be expressed in terms of the unknown torques by using the load-displacement relationship, $\phi = TL/JG$. Here there are three regions of the shaft where the internal torque is constant. On the free-body diagrams in Fig. 5–23*c* we have shown the internal torques acting on the left segments of the shaft which are sectioned in each of these regions. This way the internal torque is only a function of T_B. Using the sign convention established in Sec. 5.4, we have







(c)

$$\frac{-T_B(0.2\text{ m})}{JG} + \frac{(800 - T_B)(1.5\text{ m})}{JG} + \frac{(300 - T_B)(0.3\text{ m})}{JG} = 0$$

so that

$$T_B = 645\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Using Eq. 1,

$$T_A = -345\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{T}_A$ acts in the opposite direction of that shown in Fig. 5–23*b*.

Ex:-The shaft shown in Fig. 5–24 *a* is made from a steel tube, which is bonded to a brass core. If a torque of ***T*= 250 lb.ft** is applied at its end, plot the shear-stress distribution along a radial line of its cross-sectional area. Take $G_{st} = 11.4\left(10^3\right) ksi,\ G_{br} = 5.20\left(10^3\right) ksi$.

SOLUTION

Equilibrium.

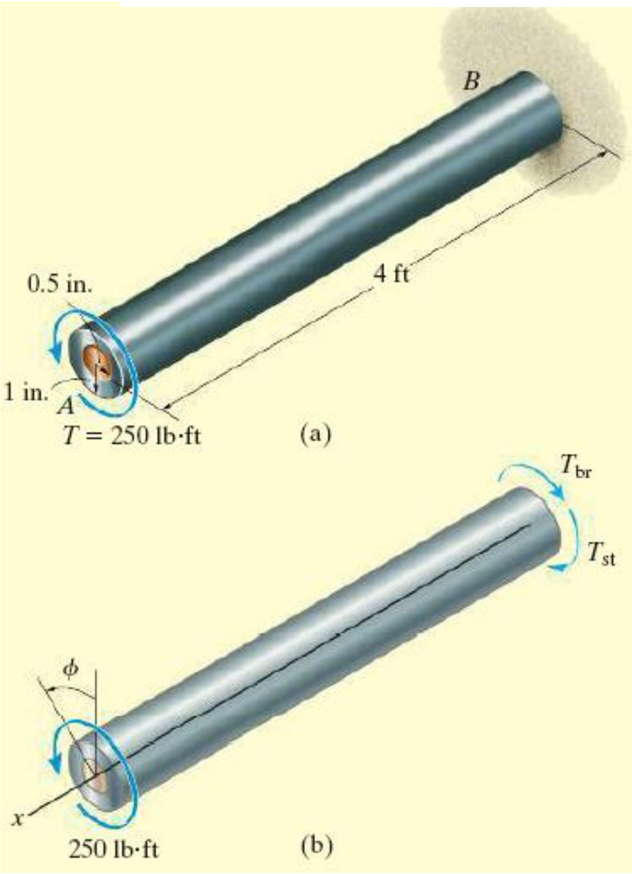
$$-T_{st} - T_{br} + (250\ \text{lb}\cdot\text{ft})(12\ \text{in./ft}) = 0 \qquad (1)$$

Load-Displacement. Applying the load–displacement relationship, $\phi = TL/JG$,

$$\frac{T_{st}L}{(\pi/2)[(1\ \text{in.})^4 - (0.5\ \text{in.})^4]\ 11.4(10^3)\ \text{kip/in}^2} =$$

$$\frac{T_{br}L}{(\pi/2)(0.5\ \text{in.})^4\ 5.20(10^3)\ \text{kip/in}^2}$$

$$T_{st} = 32.88\,T_{br} \qquad (2)$$



Strength of Materials: Second Class

Solving Eqs. 1 and 2, we get

$$T_{st} = 2911.5 \text{ lb}\cdot\text{in.} = 242.6 \text{ lb}\cdot\text{ft}$$
$$T_{br} = 88.5 \text{ lb}\cdot\text{in.} = 7.38 \text{ lb}\cdot\text{ft}$$

The shear stress in the brass core varies from zero at its center to a maximum at the interface where it contacts the steel tube. Using the torsion formula,

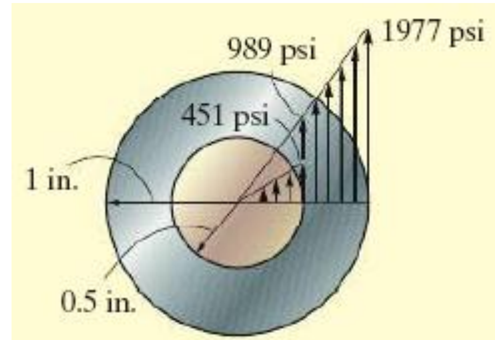
$$(\tau_{br})_{max} = \frac{(88.5 \text{ lb}\cdot\text{in.})(0.5 \text{ in.})}{(\pi/2)(0.5 \text{ in.})^4} = 451 \text{ psi}$$

For the steel, the minimum and maximum shear stresses are

$$(\tau_{st})_{min} = \frac{(2911.5 \text{ lb}\cdot\text{in.})(0.5 \text{ in.})}{(\pi/2)[(1 \text{ in.})^4 - (0.5 \text{ in.})^4]} = 989 \text{ psi}$$

$$(\tau_{st})_{max} = \frac{(2911.5 \text{ lb}\cdot\text{in.})(1 \text{ in.})}{(\pi/2)[(1 \text{ in.})^4 - (0.5 \text{ in.})^4]} = 1977 \text{ psi}$$

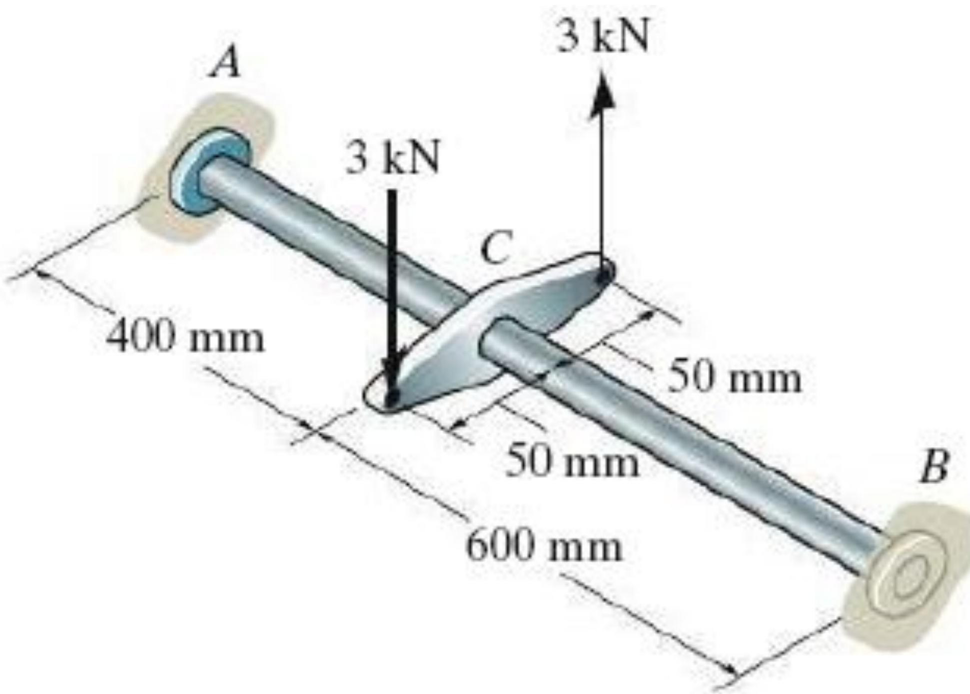
The results are plotted in Fig. 5–24 *c*. Note the discontinuity of *shear stress* at the brass and steel interface. This is to be expected, since the materials have different moduli of rigidity



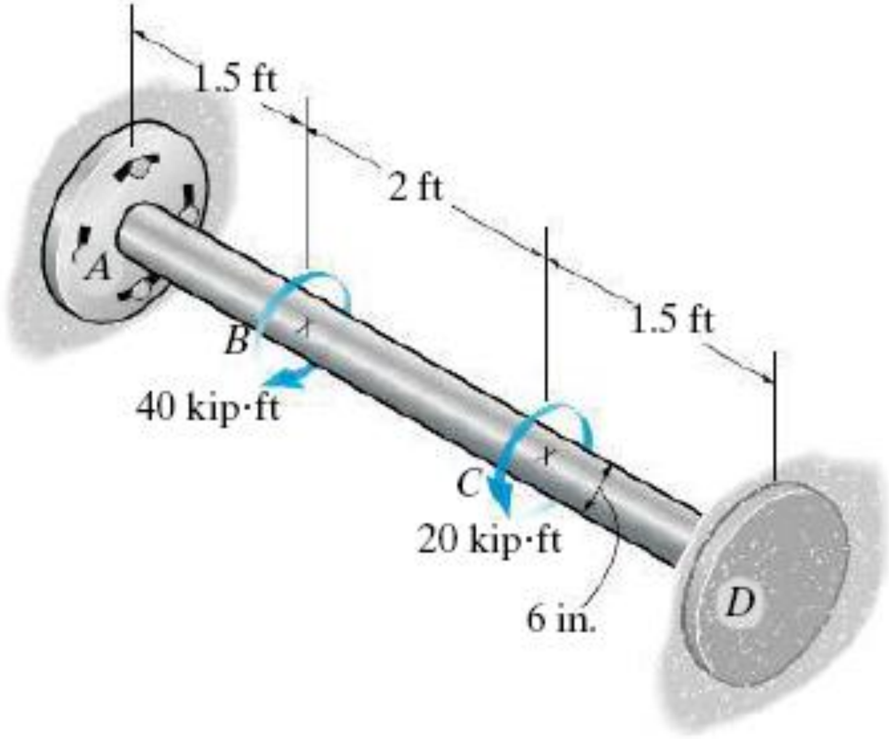
Shear–stress distribution

(c)

1- The steel shaft has a diameter of **40** mm and is fixed at its ends A and **B** . If it is subjected to the couple determine the maximum shear stress in regions **AC** and **CB** of the shaft. $\boldsymbol{G_{st}}$ **= 75 GPa.**



2:- The shaft is made of **A-36** steel and is fixed at end **D** , while end A is allowed to rotate **0.005** rad when the torque is applied. Determine the torsional reactions at these supports.



5.8 Stress Concentration

The torsion formula, , cannot be applied to regions of a shaft having a sudden change in the cross section.

The shear-stress and shear-strain distributions in the shaft become complex and can be obtained only by using experimental methods or possibly by a mathematical analysis based on the theory of elasticity.

Three common discontinuities of the cross section that occur in practice are shown in Fig. 5–31.

1) They are at *couplings* , which are used to connect two collinear shafts together, Fig. 5–31 *a.*


(a)

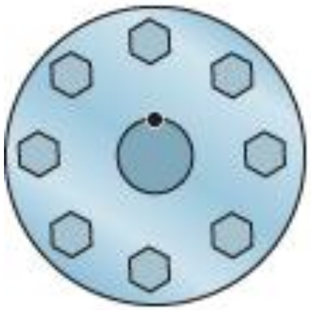
2) *keyways* , used to connect gears or pulleys to a shaft, Fig. 5–31 *b* .


(b)

3) a step shaft which is fabricated or machined from a single shaft, Fig. 5–31 *c* .


(c)

In each case the maximum shear stress will occur at the point (dot) indicated on the cross section.



This maximum shear stress can be determined from *torsional stress-concentration factor*, *K*

K is usually taken from a graph based on experimental data.

Find geometric ratio *D/d* for appropriate curve Once abscissa *r/d* calculated, value of *K* found along ordinate

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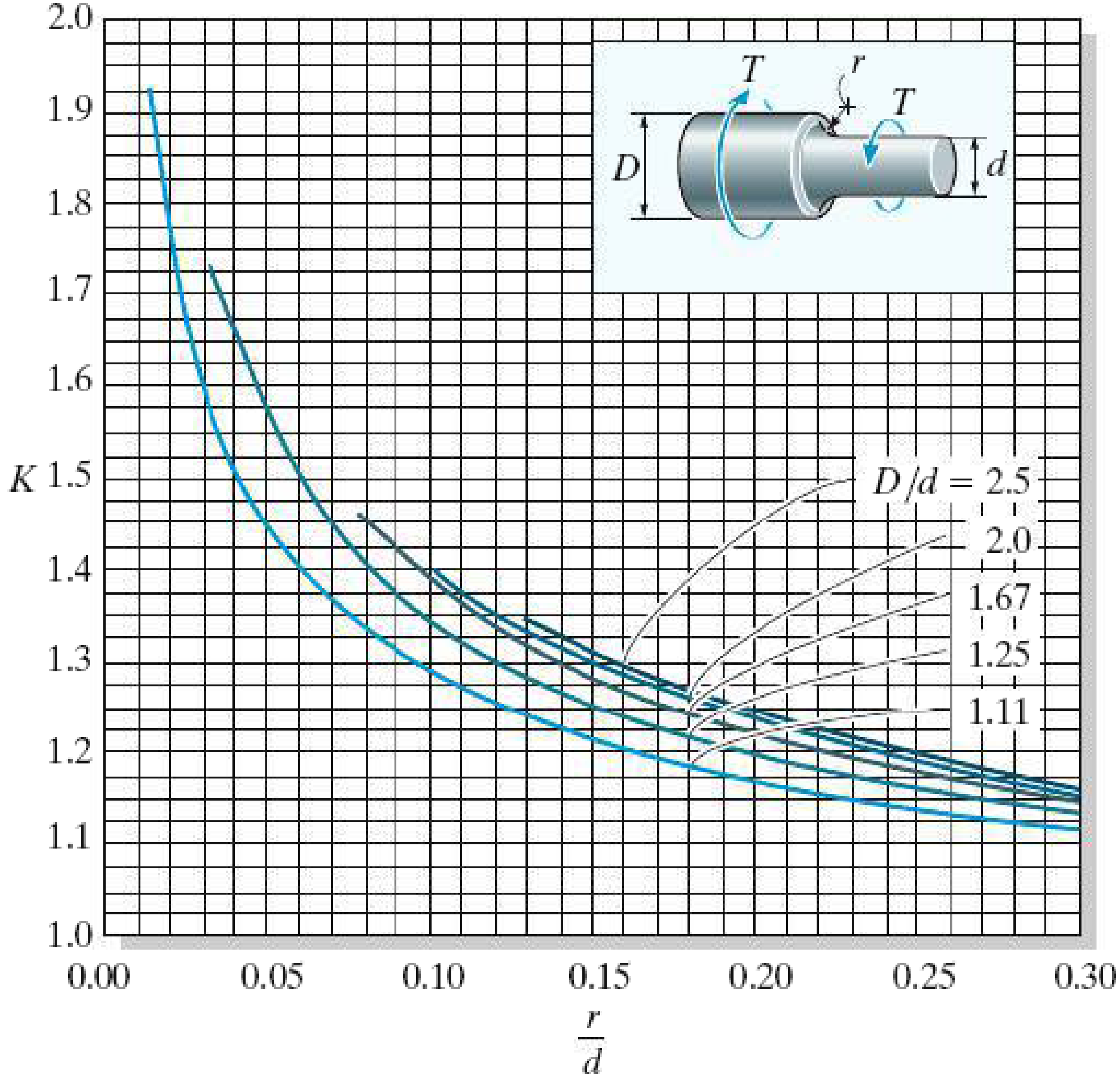
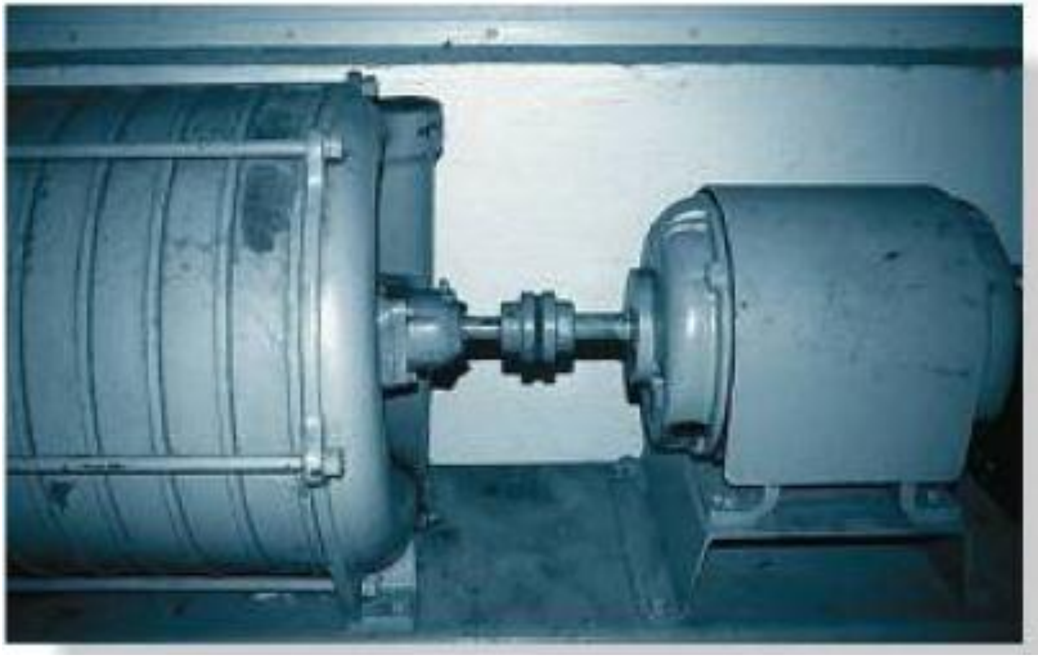


Maximum shear stress is then determined from

$$\tau_{\max} = K(Tc/J)$$



Stress concentrations can arise at the coupling of these shafts, and this must be taken into account when the shaft is designed.